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*Accounting for Higher Moment Betas in Asset –
Liability Management: Finding the Optimal Equity
Diversifier*

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Abstract

In this paper we utilize higher moment betas to identify suitable equity diversifiers for the equity part of a traditional bond- and equity portfolio. With a Modified VaR optimization program put into an Asset – Liability Management framework we show that an allocation to hedge funds can reduce the down side risk in the portfolio and at the same time improve returns.

1. Introduction

According to traditional finance theory a portfolio with a 60/40 split between bonds and stocks provides for an optimal portfolio and this has therefore been the starting position for institutional investors for decades. Even today this is a common portfolio structure, for example among pension funds. As long as the equity market is in a healthy state no problems will occur with this strategy, the problems start when the equity market no longer behaves as planned. This could be seen during 2001 to 2004 when the equity market fell heavily and left many pension funds under-funded with solvency ratios well below 100%.

Clearly, the concept of managing the asset side of the portfolio on a relative basis, often measured as tracking error against a benchmark, and having the liabilities measured in absolute terms, in form of pension pay-outs, is a risky strategy. The trend in the pension industry is towards a greater awareness of the problems inherited in this form of portfolio management and more and more pension funds therefore look for ways to move away from a pure relative benchmark approach into a structure where the asset side is managed in accordance with the liability side, so called Asset – Liability Management (ALM). As a result, other types of investments are becoming more popular, among them hedge funds. This segment of the investment universe can be a valuable tool from a pension fund's point of view and in this paper will show how they can be used in an ALM frame work to minimize the risk of ending up in the same under-funded situation as was the case in the after-math of the dot.com bubble.

Another driving factor behind how pension funds and insurance companies will manage their portfolios in the future is the upcoming changes in regulations, for example the implementation of the Basel II accord in 2007. This will put pressure on these groups to improve their risk management and to move away from a traditional bond – equity approach to a structure where the main objective is to meet future liabilities, not maximize return.

The rest of this paper is structured as follows. In section 2 we briefly describe traditional ALM techniques, in section 3 we describe alternative ALM techniques and section 4 contains an overview of higher moment diversification benefits. In section 5 we present empirical results from the study conducted, in section 6 we identify the optimal portfolio through a Modified Value-at-Risk (VaR) optimizer program and the paper ends with conclusions in section 7.

2. Traditional ALM Techniques

Until recently the focus of academic research, as well as among institutional investors, has been on the management of interest rate risk in bond- equity portfolios. Techniques as those described below have been used for decades and have gradually increased in sophistication and complexity. We here briefly touch upon some of these techniques before turning to the other part of the traditional 60/40 pension portfolio, the often tracking error managed equity part.

Cash flow matching

When a fully hedged position is desired in form of the cash flows on the asset side fully reflecting the future payouts on the liability side a perfect matching technique can be implemented. Here, after estimating the factors impacting the liability side, including mortality rates etc, an instrument is sought after that perfectly matches the liability cash flow stream. In theory, the optimal instrument is an inflation linked zero-coupon bond with the exact same cash flow stream and maturity as the liability. With this kind of instrument the interest rate risk is perfectly matched against the liability side. However, in practice is it usually hard to find an instrument that exactly fulfils the above mentioned criteria, often resulting in an asset- liability mismatch.

Immunitization

When this kind of mismatch exists from a non-perfect hedge instrument in the cash flow matching process a technique called immunization is often used. Immunization was introduced already some fifty years ago in a paper by Redington (1952), where the technique is defined as when “the existing business is immune to a general change in interest rates”. Here, the remaining interest risk is managed according to interest rate risk management techniques, for example duration matching, to hedge and optimize for different interest rate moves. From a pension fund’s perspective the present value of the future liabilities grows as interest rates fall and if the growth on the asset side is less this can result in a situation of under-funding.

As pointed out by Martellini (2006), the perfect matching technique, as well as the immunization technique, is similar to investing only in the risk-free rate in the CAPM framework, thus providing a perfect, or near perfect, hedge but at a high alternative cost in form of a non-optimal risk-return profile.

3. Alternative ALM Techniques

Standard Core - Satellite approach

Many pension funds still have a relative return approach to investing where the performance of the portfolio is evaluated compared to a benchmark. This is often done within a tracking error framework where limits are set of how much the standard deviation is allowed to deviate from the chosen benchmark. The core-satellite approach splits the portfolio in two parts, a passive index invested part i.e. the core portfolio and an active part, the satellite. The idea is to use a low cost index-replicating product as the core and a higher tracking error product as the satellite. Compared to implementing just one active mandate for the management of the whole portfolio a total lower cost can be achieved, while keeping the total tracking error of the portfolio at the same level. A further development of this area is the concept of portable alpha. Here the core portfolio is structured as a passive low-tracking error portfolio, as described earlier, while the satellite portfolio is structured to isolate and facilitate the transfer of “pure alpha” into the portfolio. The satellite portfolio is thereby managed on an absolute return basis and not on a relative benchmark basis that has been the standard historically. The alpha has generally found in hedge fund strategies with a low beta to the benchmark index used in the core part of the portfolio, for example equity market neutral.

An alternative way of extracting alpha is presented by Fung and Hsieh (2004). Their approach builds on the research done in what is named Asset based style factors models. Here, underlying risk factors in form of frequently traded instruments are used to explain the return on a specific hedge fund strategy. This can facilitate a deeper understanding on what is driving the return for hedge funds and furthermore, provide more room for quantitative analysis since longer time series are available. Fung and Hsieh (2004) use Equity Long/Short hedge funds as an alpha generator and the idea is to isolate the alpha the managers create via an elimination of systematic risk factors. After a regression procedure they conclude that two of the factors included in the factor model presented by Fama-French (1992) are statistically significant systematic risk factors. These are the excess return of the market, express as the market return in excess over the risk free rate, and the performance difference between small and large capitalization stocks. The strategy is then to extract the pure alpha generated by Equity Long/Short hedge fund managers by going short the two systematic risk factors and going long the Equity Long/Short hedge fund index. Left is then the alpha-part which can be used as part of the satellite portfolio in the core – satellite frame work. According to the authors this strategy can provide for an improved risk-return profile compared to an investment in the broad equity market.

Dynamic Core- Satellite approach

The strategy of the standard core – satellite approach described above is often implemented with static weights between the core and the satellite. A more optimal solution is proposed by Amenc, Malaise and Martellini (2004) where the surplus of the portfolio over the benchmark is managed in a dynamic way. The idea here is to allocate more to the risky satellite portfolio when the overall portfolio has outperformed the benchmark and less when an underperformance has occurred. This leads to higher probability of an out-performance and a decreased probability of an underperformance.

Although being an interesting extension of the standard approach, it works best during normal market conditions. Since the satellite portfolio normally has a high equity concentration, and this part is given more weight after a period of out-performance, the portfolio will be vulnerable to stock market crash-like scenarios. To manage to effectively rebalance in such a scenario is not likely and the result might be a severe erosion of the over-performance, or even worse, result in a solvency ratio well below 100%, as seen in many pension funds in the aftermath of the dot.com bubble.

4. Higher Moment Diversification Benefits in ALM

Minimizing down side risk with Equity Diversifier

The approaches we have described above are generally designed in a framework where risk is measured only by the standard deviation. This follows the standard approach seen in the CAPM and the mean-variance asset allocation approach. Implementing a similar approach will be problematic when hedge funds, and many traditional assets as well, are included. As described in Haglund (2006) among others, the returns for different hedge fund indexes display non-normal return distributions when longer time frames are studied. Therefore, working with a framework that assumes that asset returns are normally distributed can underestimate down-side risk and lead to non-optimal portfolio allocations. The standard deviation only measures deviations from the mean and puts equal weight to positive and negative deviations from the mean. Usually preferences are non-symmetrical in that the perceived utility of a positive result is often less than the impact of a negative result of equal magnitude. A way of accounting for this preference structure and account for non-normal distributions is to introduce Bond and Equity diversifiers that incorporate higher moment betas.

The concept of Bond and Equity diversifiers was introduced by Martellini and Ziemann (2005) and build upon the three moment CAPM, see for example Jurczenko and Maillet (2001), and the four moment CAPM and higher moment betas described in Favre and Rinaldo (2003). Here, higher moment diversification benefits, i.e. a reduction in the variance, skewness and kurtosis, are calculated to determine appropriate hedge fund

strategies to include in the portfolio. The aim is to reduce the downside risk of the portfolio. Suitable indexes acting as diversifiers for the equity part of the portfolio is named Equity diversifiers and equally for the bond part they are named Bond diversifiers. The following formulas are used to calculate the higher moment betas:

$$Co - Variance Beta = \frac{E\left[\left(R_i - \overline{R}_i\right)\left(R_m - \overline{R}_m\right)\right]}{E\left(\left(R_m - \overline{R}_m\right)^2\right)} \quad (1)$$

$$Co - Skewness Beta = \frac{E\left[\left(R_i - \overline{R}_i\right)\left(R_m - \overline{R}_m\right)^2\right]}{E\left(\left(R_m - \overline{R}_m\right)^3\right)} \quad (2)$$

$$Co - Kurtosis Beta = \frac{E\left[\left(R_i - \overline{R}_i\right)\left(R_m - \overline{R}_m\right)^3\right]}{E\left(\left(R_m - \overline{R}_m\right)^4\right)} \quad (3)$$

where R_i is the return on portfolio i and R_m is the return on benchmark (market) portfolio. A value below 1 for the Co-variance beta and the Co-kurtosis beta indicates diversification benefits, while a value below 1 for the Co-skewness beta indicates diversification benefits when the portfolio exhibits a negative skewness and a value above 1 when there is a positive skewness.

5. Empirical Results

Here we build upon the research conducted by EDHEC Risk and Management Research Centre and presented in, among others, the papers of Martellini and Ziemann (2005) and Martellini (2006).

To find the optimal equity diversifiers we use historic return figures for the sub indexes of the CSFB/Tremont Hedge Index ranging from January 1994 – February 2006. The period studied includes several types of market environments and a few periods of severe market stress. To end up with more robust estimates of the higher moments diversification benefits we use a longer time frame than the October 2001 to September 2004 period used in Martellini and Ziemann (2005). To get a wider range of possible diversifiers we study the higher moments of 10 of the sub strategies of the CSFB/Tremont Hedge Index. The rationale for not using the investable sub index of CSFB/Tremont Hedge Index is that the non-investable is more representative as a benchmark for hedge fund performance since a selection process precedes the construction of the investable index.

Return distributions

We start with a calculation of some return distribution statistics, in form of skewness and kurtosis ², for the different sub indexes.

$$Skewness_p = \sum \left(\frac{x_{p,i} - \bar{x}_p}{\sigma_p} \right)^3 \frac{n}{(n-1)(n-2)} \quad (4)$$

$$Kurtosis_p = \sum \left(\frac{x_{p,i} - \bar{x}_p}{\sigma_p} \right)^4 \frac{n(n+1)}{(n-1)(n-2)(n-3)} - \frac{3(n-1)^2}{(n-2)(n-3)} \quad (5)$$

where x_p is the return on portfolio p , \bar{x}_p is the mean return on the portfolio, σ_p is the standard deviation and n is the number of observations.

Following that we conduct a test of normality with the Bera-Jarque test. We here calculate the Bera-Jarque statistic at the 95% confidence level, according to the formula below³.

$$Bera - Jarque = \frac{n}{6} \left[Skewness^2 + \frac{Kurtosis^2}{4} \right] \quad (6)$$

where n is number of observations, and skewness and kurtosis as in formula (4) and (5).

Exhibit 1a Return profile for CSFB/Tremont sub indexes

	Hedge Index	Fund Convertible Arbitrage	Equity Neutral	Mkt	Event Driven	Distressed
Skewness	0.089	-1.327	0.304		-3.430	-2.912
Kurtosis	2.317	3.078	0.316		24.365	19.052
Bera-Jarque test	33.31	101.89	2.90		3951.06	2447.51
Normality	Rejected	Rejected	Not rejected		Rejected	Rejected

Exhibit 1b Return profile for CSFB/Tremont sub indexes

	Risk Arbitrage	Fixed Income Arb	Global Macro	Long/Short Equity	Managed Futures
Skewness	-1.275	-3.063	0.008	0.202	0.015
Kurtosis	6.673	16.348	2.887	3.924	0.384
Bera-Jarque test	314.69	1879.50	51.39	95.94	0.91
Normality	Rejected	Rejected	Rejected	Rejected	Not rejected

² Skewness and kurtosis are here zero subject to a normal distribution.

³ The critical value of the Bera-Jarque test is 5.99 at the 95% confidence level, i.e. a value below 5.99 rejects the hypothesis that the return is non-normal and instead assumes that the return is normally distributed.

As displayed in Exhibit 1a and 1b, the only two indexes where the returns can be assumed to be normally distributed are Equity Market Neutral and Managed Futures. All other indexes show various degree of non-normal distributions, where a big risk of large negative observations, seen in form a large negative skewness and a large value of kurtosis, exists in Event Driven, Distressed, Risk Arbitrage and Fixed Income Arbitrage.

In Exhibit 2 we present the same statistics for the equity portfolio we are diversifying, i.e. MSCI World. Against what is a common assumption about the broad equity market, MSCI World does not display a return series that is normally distributed for the studied time period.

Exhibit 2 Return profile for MSCI World

	Skewness	Kurtosis	Bera-Jarque test	Normality
MSCI World	-0.675	0.880	16.00	Rejected

Higher moment betas

To determine which of the sub indexes that is suitable to use as equity diversifier for the equity part in a traditional portfolio of stocks and bonds we calculate the higher moment betas according to formula (1) to (3) described above.

Exhibit 3a Higher moment betas with MSCI World for CSFB/Tremont sub indexes

	Hedge Fund Index	Convertible Arbitrage	Equity Neutral	Mkt	Event Driven	Distressed
Co-Variance beta	0.28	0.05	0.08		0.25	0.27
Co-Skewness beta	0.45	0.14	0.05		0.51	0.58
Co-Kurtosis beta	0.30	0.11	0.07		0.39	0.42

Exhibit 3b Higher moment betas with MSCI World for CSFB/Tremont sub indexes

	Risk Arbitrage	Fixed Income Arbitrage	Global Macro	Long/Short Equity	Managed Futures
Co-Variance beta	0.14	0.01	0.16	0.47	-0.08
Co-Skewness beta	0.27	0.16	0.34	0.56	-0.51
Co-Kurtosis beta	0.23	0.04	0.15	0.50	-0.31

Due to the negative skewness seen in MSCI World, according to Exhibit 2, we are looking for a value below 1 in all the higher moment betas presented in Exhibit 3a and 3b as an indication of higher moment diversification benefits.

*Exhibit 4 Ranking of higher moment diversification benefits**

CSFB/Tremont sub index	Variance diversifier	Skewness diversifier	Kurtosis diversifier
Hedge Fund Index	9	7	7
Convertible Arbitrage	3	3	4
Equity Mkt Neutral	4	2	3
Event Driven	7	8	8
Distressed	8	10	9
Risk Arbitrage	5	5	6
Fixed Income Arbitrage	2	4	2
Global Macro	6	6	5
Long/Short Equity	10	9	10
Managed Futures	1	1	1

* Ranking from 1 to 10, where 1 indicates the best moment diversifier and 10 the worst.

In Exhibit 4 and 5 we summarize the results from the calculation of the higher moment betas for each sub index. As displayed, Managed Futures demonstrates the best values of higher moment diversification benefits and therefore ranks the highest. The other top candidates are Fixed Income Arbitrage, Equity Market Neutral and Convertible Arbitrage. The sub indexes with all higher moment betas lower than 0.16 are classified as an equity diversifier.

Exhibit 5 Equity diversifiers

CSFB/Tremont sub index	Equity diversifier
Hedge Fund Index	No
Convertible Arbitrage	Yes
Equity Mkt Neutral	Yes
Event Driven	No
Distressed	No
Risk Arbitrage	No
Fixed Income Arbitrage	Yes
Global Macro	No
Long/Short Equity	No
Managed Futures	Yes

6. Higher Moment Portfolio Optimization

After identifying appropriate equity diversifiers we use these sub indexes to construct an optimal portfolio in terms of minimizing downside risk for the equity part of the portfolio. The equity diversifiers we use are Convertible Arbitrage, Equity Market Neutral and Managed Futures. Although fulfilling the criteria set for an equity diversifier, we exclude Fixed Income Arbitrage because too close connection in terms of risk factors with the bond part of the portfolio.

To take into account the non-normal distribution patterns we identified in Exhibit 1a and 1b we use a Modified VaR approach to get the optimal portfolios, using restrictions of 0%, 5%, 15%, 25% and 35% of the portfolio invested in hedge fund indexes. Furthermore, we set a constraint of maximum 40% of each single hedge fund index in the diversifier. The restrictions and weight constraints are the same as in Martellini and Ziemann (2005). The Modified VaR is a version of VaR that accounts for skewness and kurtosis in the form of a Fisher-Cornish extension of the Gaussian VaR, as suggested by Favre and Galeano (2002). The modified VaR is then modified from formula (7) into formula (8).

$$VaR_i = E(R_i) + z_c \cdot \sigma_i \quad (7)$$

$$Mod VaR_i = E(R_i) + \left(z_c + \frac{z_c^2 - 1}{6} S_i + \frac{z_c^3 - 3z_c}{24} K_i - \frac{2z_c^3 - 5z_c}{36} S_i^2 \right) \sigma_i \quad (8)$$

where z_c depends of the level of confidence interval ($z_c = -1.96$ at the 95% confidence level), and S_i and K_i are the skewness and kurtosis of portfolio i and σ_i is the standard deviation of the portfolio. S_i and K_i are as defined in formula (4) and (5). If the distribution is normal S_i and K_i are equal to zero and we are back at the same equation as for the parametric Gaussian VaR, formula (7).

To get a risk-adjusted return measure we use the modified Sharpe-ratio which, unlike the normal Sharpe, accounts for skewness and kurtosis of the distribution. In this way we account for the non-normal return distribution seen in several of the indexes.

$$Modified\ Sharpe - ratio = \frac{R_p - R_F}{VaR_{Mod}} \quad (9)$$

where R_p is the return on portfolio p , R_F is the risk-free rate and VaR_{Mod} is the Modified VaR of portfolio p .

Exhibit 6 Return statistics with different weights of equity diversifier

Allocation to equity diversifier	0%	5%	15%	25%	35%
Annualized return	6.79%	6.94%	7.23%	7.50%	7.74%
Yearly volatility	13.89%	13.21%	11.85%	10.53%	9.25%
95% Modified VaR	-8.49%	-8.00%	-7.02%	-6.06%	-5.13%
Skewness	-0.675	-0.657	-0.613	-0.554	-0.474
Kurtosis	0.88	0.810	0.648	0.452	0.221
Bera-Jarque test	16.00	14.69	11.85	8.83	5.85
Sharpe ratio*	0.20	0.22	0.27	0.33	0.40
Modified Sharpe ratio*	0.034	0.037	0.043	0.051	0.062

* Calculated with a risk free rate of 4.1%

After running the Modified VaR optimizer with the constraints detailed above we reach the results displayed in Exhibit 6. As can be seen, the inclusion of the equity diversifier in the portfolio has several positive effects. The annualized return is increasing while the volatility is decreasing as we allocate more and more to hedge funds, resulting in higher Sharpe ratios. Notable is also the reduction in skewness and kurtosis, with a 35% allocation to the equity diversifier the return series fulfils the criteria for being normally distributed according to the Bera-Jarque test. This reduction in negative tail-risk is seen in form of a sharp improvement of the Modified VaR figure and therefore also in higher Modified Sharpe ratios.

Exhibit 7 Improvement in annualized return

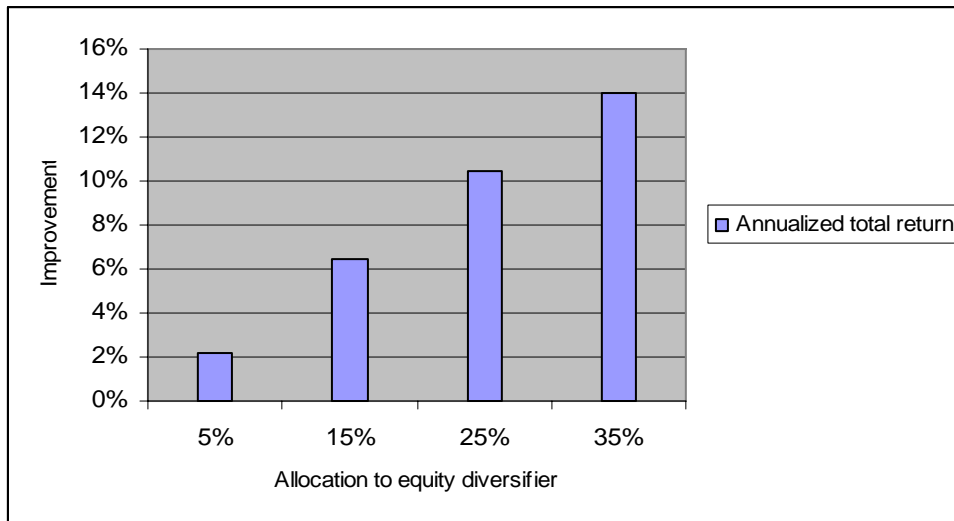
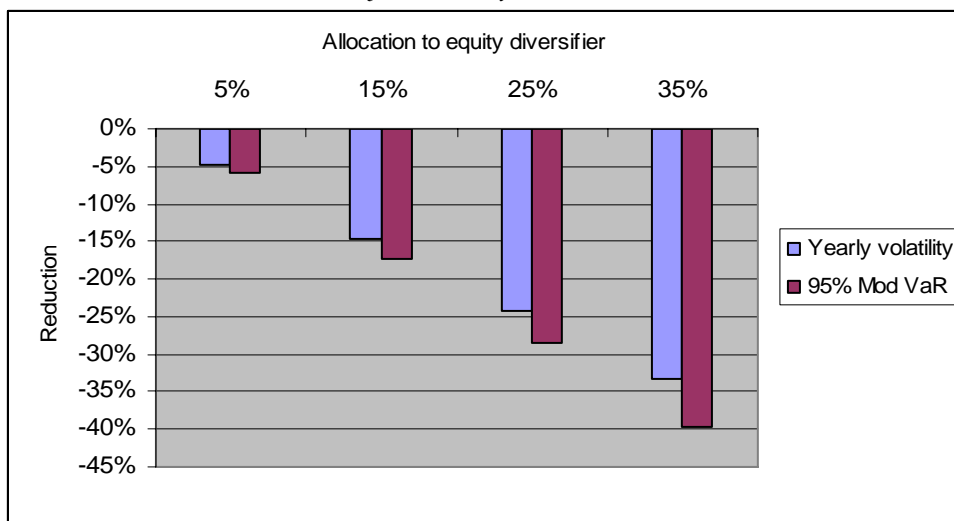


Exhibit 8 Reduction in volatility and Modified VaR

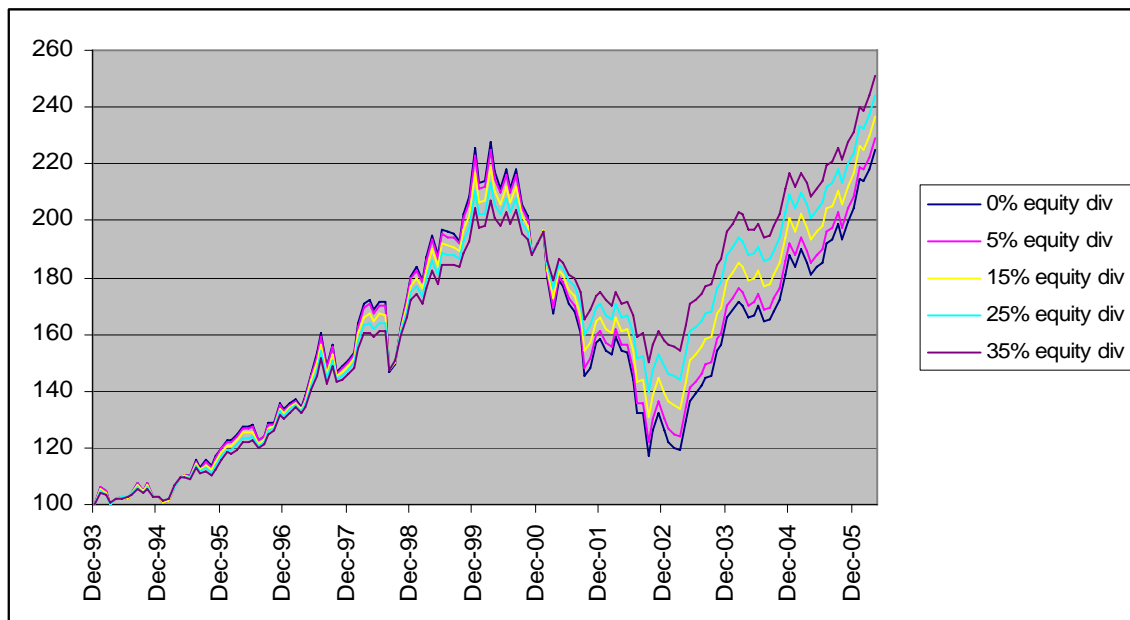


The improvement in annualized return is presented in Exhibit 7 and the improvement range from 2% to 14%. With an allocation to hedge funds of 15%, a reasonable level for a pension fund, the annualized return goes from 6.79% to 7.23%, an improvement of 6.5%. The same allocation results in a change in volatility, Exhibit 8, from 13.89% to 11.85%, a reduction of 14.7%, and a change in the Modified VaR from -8.49% to -7.02%, a reduction of 17.3%.

Compared to the study by Martellini and Ziemann (2005) the improvement in annualized mean return with 15% allocated to the equity diversifier is less than the 35% increase they identified for the period October 2001 to September 2004. The reduction in volatility and Modified VaR is of an equal magnitude, circa 17%. The lower increase in return found here is likely to be partly a result of the longer period used, which includes several periods of adverse market conditions.

Finally, we display the rebased performance of the portfolio with different allocations to the equity diversifier in Exhibit 9. In addition to the positive effects on annualized return and down side risk discussed earlier the maximum drawdown is also greatly reduced when the allocation increases to hedge funds. The portfolio without hedge funds suffers from a 47.8% drawdown, while the portfolio with 35% in hedge funds only displays a 27.6% drawdown. Having that said, had pension funds allocated more to hedge funds during the last 10 years the problems seen in solvency ratios being far below 100% during 2000 – 2004 would have been far less severe.

Exhibit 9 Rebased performance of portfolio with different weights in equity diversifier



7. Conclusions

After the solvency problems seen in recent years at many pension funds more attention is directed to an Asset – Liability Management frame work where the total risk of the portfolio is considered. The management of the interest rate risk in pension portfolios have been managed with interest rate management techniques for some time but the equity part of the portfolio has normally been managed in a simplistic tracking error environment where risk is defined only as deviations from the benchmark. The trend is towards a management of the portfolios on an absolute return basis, where the asset side in a better way matches the growth on the liability side in terms of pension payouts.

By the implementation of an equity diversifier constructed from different hedge fund styles we show that the return can be improved and the risk, incorporating higher moment of the distribution, can be reduced significantly when added to the passive index part of a pension portfolio. An allocation to the appropriate hedge fund styles can in this way be used as a tool in an Asset – Liability Management frame work where one of the main goals is to avoid situations of the pension plan being under-funded.

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